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| **Math 260 - Week 2 Lab Name:** |  |
| As we have seen, using $\lim\_{h\to 0}\frac{f\left(x+h\right)-f(x)}{h}$ to find a derivative each time is a long process and often cumbersome. But, there are a few *algorithms* that work perfectly for finding derivatives. They are **the power rule, the product rule, the quotient rule, and the chain rule.** -------------------------------------------------------------------------------------------------------------------------------------------------------------**Category 1: Power Rule****Directions:** Look at the examples below then answer questions 1 & 2. 1. $f\left(x\right)=x^{3}-3x+4$ b.) $f\left(x\right)=5x^{4}$ c.) $f\left(x\right)=3\sqrt{x}$

 $f^{'}\left(x\right)=3x^{2}-3$ $f^{'}\left(x\right)=20x^{3}$ $f^{'}\left(x\right)=\frac{3}{2\sqrt{x}}$ d.) $f\left(x\right)=\frac{6}{x^{4}}$  $f^{'}\left(x\right)=-\frac{24}{x^{5}}$1. Describe in your own words how to find a derivative using the Power Rule. What must be done to c.) and d.) above before using the power rule to differentiate ?
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| 2.) Find the derivative of  |
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| **Category 2: Product Rule****Directions:** Look at the examples below then answer questions 3. 1. $f\left(x\right)=\left(x^{2}+4x-2\right)(5x-7)$

 $f^{'}\left(x\right)=5\left(x^{2}+4x-2\right)+\left(2x+4\right)\left(5x-7\right)=5x^{2}+20x-10+10x^{2}-14x+20x-28=$ $= 15x^{2}+26x-38$ ← **SIMPLIFIED ANSWER**1. $f\left(x\right)=2x\sqrt{x}$

 $f^{'}\left(x\right)=\left(2x\right)\frac{1}{2\sqrt{x}}+2\left(\sqrt{x}\right)=\frac{x}{\sqrt{x}}+2\sqrt{x}=\sqrt{x}+2\sqrt{x}=3\sqrt{x}$ ↑ **RULE** ↑ **answer** ↑ **SIMPLIFIED**  3.) i.) Describe, in your own words, how to find a derivative using the product rule.ii.) Find the derivative for f(x) = (2x + 3)(5x2 – 3x + 1)  iii.) Explain what steps were taken to change the answer to the simplified form for b.) |
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| **Category 3: Quotient Rule****Directions:** Look at the examples below then answer the questions 4.  $f\left(x\right)=\frac{x}{x+3}$ b.) $f\left(x\right)=\frac{x^{3}-4}{x^{2}+5}$$ f^{'}\left(x\right)=\frac{\left(x+3\right)\left(1\right)-\left(x\right)(1)}{\left(x+3\right)^{2}}=\frac{3}{\left(x+3\right)^{2}}$  $f^{'}\left(x\right)=\frac{\left(x^{2}+5\right)\left(3x^{2}\right)-\left(x^{3}-4\right)\left(2x\right)}{\left(x^{2}+5\right)^{2}}=\frac{x^{4}+15x^{2}+ 8x}{\left(x^{2}+5\right)^{2}}$ **RULE SIMPLIFIED ANSWER**  **RULE SIMPLIFIED ANSWER**4.) Describe, in your own words, how to find the derivative using the Quotient rule, then find f ’(x) for    |
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| 5.) Research and explain the **Sum Rule** for derivatives. |
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| **Category 4: Chain Rule** **Directions:** Look at the examples below then answer the question 6 & 7. 1. $f\left(x\right)=\left(2x+1\right)^{3}$ b.) $f\left(x\right)=\sqrt{3x-1}$

 $f^{'}\left(x\right)=\left(3\right)\left(2x+1\right)^{2}(2)=6\left(2x+1\right)^{2}$ $f^{'}\left(x\right)=\left(3\right)\left(\frac{1}{2}\right)\left(3x-1\right)^{-\frac{1}{2}}=\frac{3}{2\left(3x-1\right)^{\frac{1}{2}}}=\frac{3}{2\sqrt{3x-1}}$ c.) $f\left(x\right)=\left(3x^{2}+2x\right)^{5}$ $f^{'}\left(x\right)=\left(5\right)\left(3x^{2}+2x\right)^{4}\left(6x+2\right)=10(3x+1)\left(3x^{2}+2x\right)^{4}$ **Rule Simplified Answer**6.) Describe, in your own words, how to find the derivative using the Chain Rule, then find the  derivative for  . |
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| 7.) For example b.) above, explain why the exponent is -1/2 in the first step ? Explain how the  answer to c.) has a 10 in front. |
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| . **Part 2: Higher Order Derivatives**Higher order derivatives are derivatives of derivatives. The significance of a derivative depends on what x and f(x) represent. For example, for the graph of f(x), and a point x on the graph f ’(x) is the slope of the curve, and f ’’(x) is the rate at which the slope is changing from point to point, called the curvature.If s(t) is a displacement or position function, then s ’(t) is the velocity, s ’’(t) is the acceleration, s’’’(t), s(4)(t), s(5)(t), s(6)(t), are the jerk, jounce/snap, crackle and pop. 8.) Check the box below each valid form of notation for a higher order derivative. y’’’ $\frac{d^{3}y}{dx^{3}}$ $\frac{dy^{3}}{d^{3}x}$ y’’’’ y(6) f ’’(x) |
|  [ ]  [ ]  [ ]  [ ]  [ ]  [ ]   |
| 9.) Examine the higher derivatives a, b, c below, then answer questions i and ii.1. $f\left(x\right)=x^{3}-2x^{2}+5x-7$ b.) $f\left(x\right)=3\sqrt{x}$ c.) $f\left(x\right)=\frac{5}{x^{4}}$

 $f'''\left(x\right)=6$ $f^{'''}\left(x\right)=-\frac{3}{4x^{\frac{3}{2}}}$ $f^{\left(4\right)}\left(x\right)=\frac{4200}{x^{8}}$i. For example a.) what value does every higher derivative have starting with the fourth derivative ?  |
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| ii. Does the same value occur for example b.) or c.) ? Explain why or why not ? |
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| 9.) Find the 3rd derivative for , show all work. |
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| 10.) As a dare devil is shot from a cannon. His wallet drops from his pocket when he is 70 feet from the ground. The position function of the wallet is given by s(t) = -26t2 + 73t + 70, t in seconds. Use your calculator, round all answers to the 100ths 1. How long will it take the wallet to hit the ground ?
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|  b.) What is the instantaneous velocity of the wallet when t = 2.2 seconds ?  |
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|  c.) What is the velocity of the wallet as it hits the ground ? |
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|  d.) What is the velocity of the wallet after it lands in the mud on the ground ? |
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| 11.) To find the derivative of a piecewise function, you must differentiate each piece. If  asked to find the derivative at a particular value of x, the correct derivative must be  chosen according to the domain of each piece of f(x). Find f ’(x) and f ’(-3), f ’(2), and f ’(3), f ’(10).    |

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| 12.) The energy output E of an electric device is a function of time given by the formula  E = t(1 + 3t)2. Power, P is the first derivative of E. Find the Power (in Watts) generated by the device at 3 seconds. |

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| 13.) Find the derivative of **f(r)** with respect to **r** given S and n are constant.   |
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| 14.) Find the rate of change of f(x) at x = 0:   |
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